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CS-Based CSIT Estimation for Downlink Pilot Decontamination in Multi-Cell FDD Massive MIMO

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CONTENTS

1. Introduction

- Reliable channel state information at transmitter (CSIT) is essential to fully exploit potential advantages of massive MIMO.

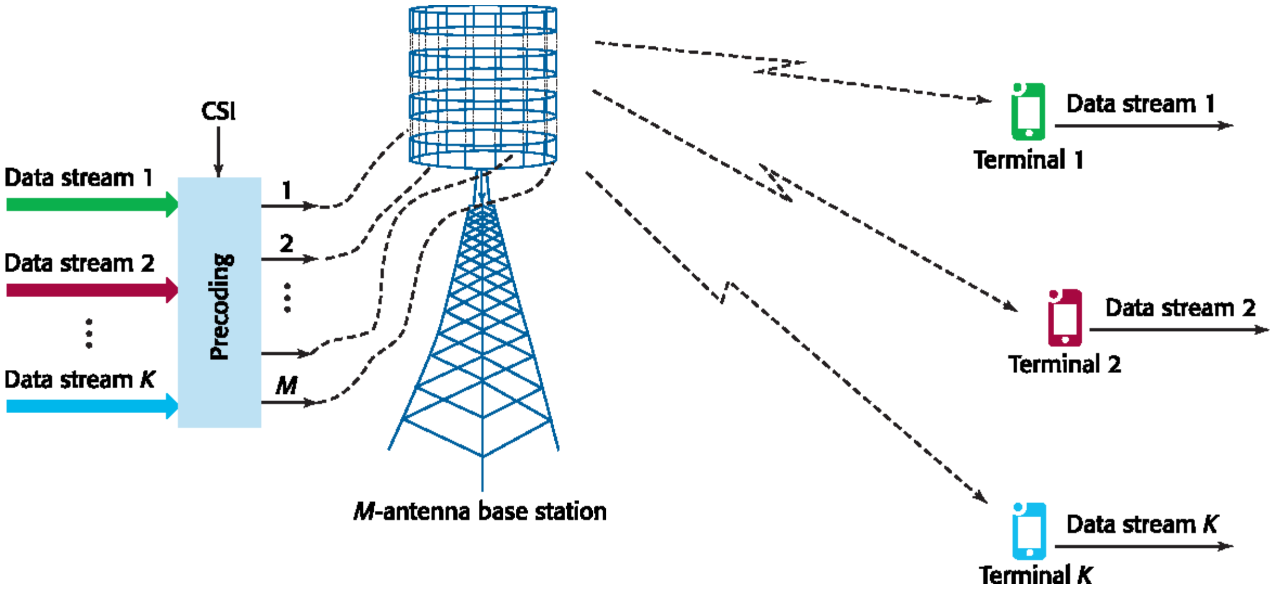


Fig. 1. Typical massive MIMO system.

1. Introduction

- However, CSIT for frequency division duplex (FDD) massive MIMO can be more challenging, since single-antenna users have to acquire and feedback the high-dimensional channels to the base station (BS)[3]-[11].

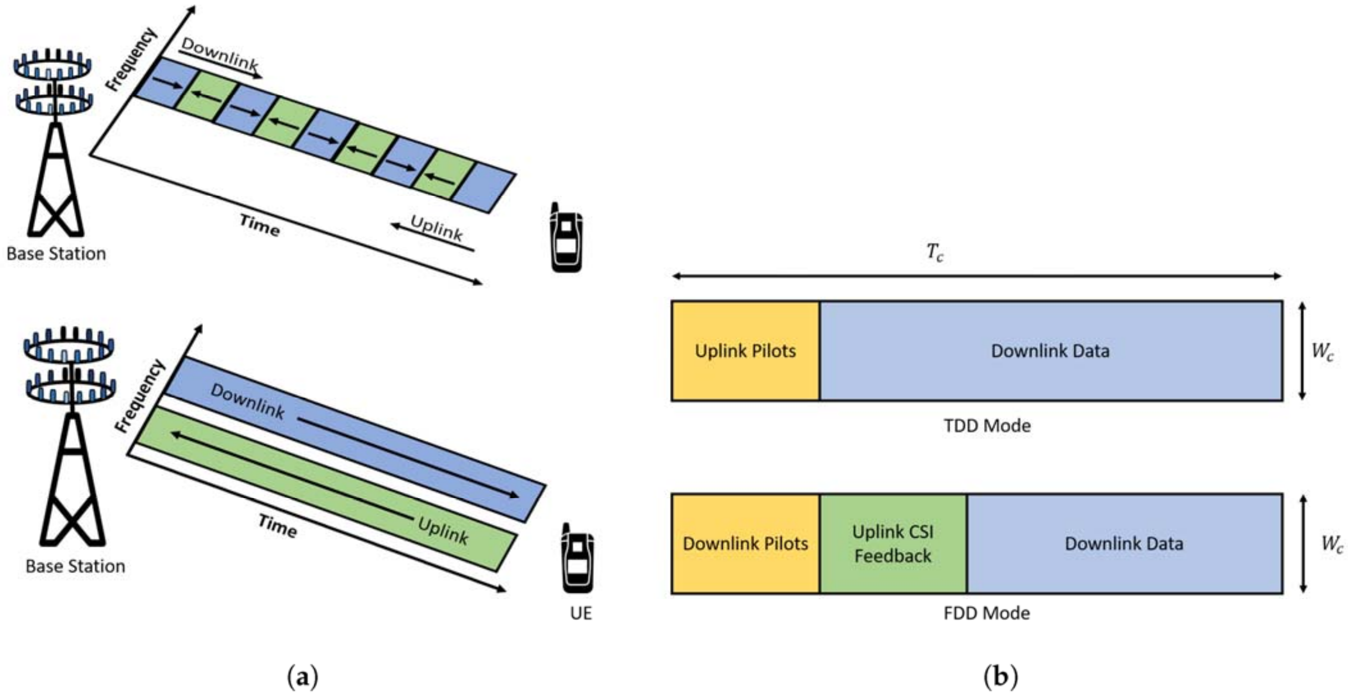


Fig. 2. (a) FDD and Time Division Duplexing (TDD) mode: Massive works best in TDD mode. (b) Typical pilot transmission and CSI feed back mechanism in FDD and TDD mode.

1. Introduction

- Several CSIT estimation schemes proposed for FDD massive MIMO[4]-[11].
  - For covariance-assisted channel estimation, it may be inaccurate to obtain downlink covariance matrix from uplink channel information[4] ;
  - The proposed methods in [5]-[7] assume the delay-domain sparsity of massive MIMO channel, which may not hold in indoor scenarios;
  - In some works, the broad-band systems are not considered[8]-[11];
  - Besides, the existing schemes [4]–[12] only consider the single-cell scenario, and they may suffer from downlink pilot contamination due to inter-cell-interference (ICI).

- In this paper, we propose a CS-based CSIT estimation scheme to alleviate the pilot contamination in multi-cell FDD massive MIMO systems.

CONTENTS

2. System Model

- Multi-cell FDD massive MIMO system composed of L hexagonal cells.
- Each cell consists of an M-antenna BS and N single-antenna users ($N \ll M$).
- For the kth user in the $\tilde{l}$ th cell, the received downlink signal of the pth subcarrier can be expressed as

$$y_{k,\tilde{l},p} = \mathbf{x}_{\tilde{l},p}^{\mathrm{T}} \mathbf{h}_{k,\tilde{l},p} + \sum_{l=0, l \neq \tilde{l}}^{L-1} \mathbf{x}_{l,p}^{\mathrm{T}} \mathbf{h}_{k,l,p} + v_{k,\tilde{l},p}, 1 \leq p \leq P, \qquad (1)$$

where $\mathbf{h}_{k,l,p} \in \mathbb{C}^{M \times 1}$ denotes the downlink channel, $\mathbf{x}_{l,p} \in \mathbb{C}^{M \times 1}$ is the transmitted signal from the l th BS, $v_{k,\tilde{l},p}$ is additive white Gaussian noise, and P is the size of one OFDM symbol.

- Two reasons for the challenges: the high dimension of $\mathbf{h}_{k,\tilde{l},p}$ and the ICI in equation (1), i.e., $\sum_{l=0, l \neq \tilde{l}}^{L-1} \mathbf{x}_{l,p}^{\mathrm{T}} \mathbf{h}_{k,l,p}$.

2. System Model

- As shown in Fig. 3, the angle-domain massive MIMO channel vectors $\tilde{\mathbf{h}}_{k,l,p} = \mathbf{F}^* \mathbf{h}_{k,l,p}$ exhibit the sparsity[10,11], where $\mathbf{F}$ is the unitary transformation matrix.
- Denoting $\Omega_{k,l,p}$ as the support of $\tilde{\mathbf{h}}_{k,l,p}$, the channels of different subcarriers are assumed to share the same sparsity pattern[11], i.e.,

$$\Omega_{k,l,1} = \Omega_{k,l,2} = \cdots = \Omega_{k,l,P} = \Omega_{k,l}. \qquad (3)$$

- For a group of K users physically close to each other, their angle-domain channels share the partially common sparsity[8], i.e.,

$$\bigcap_{k=1}^{K} \Omega_{k,l} = \Omega_c \neq \varnothing. \qquad (4)$$

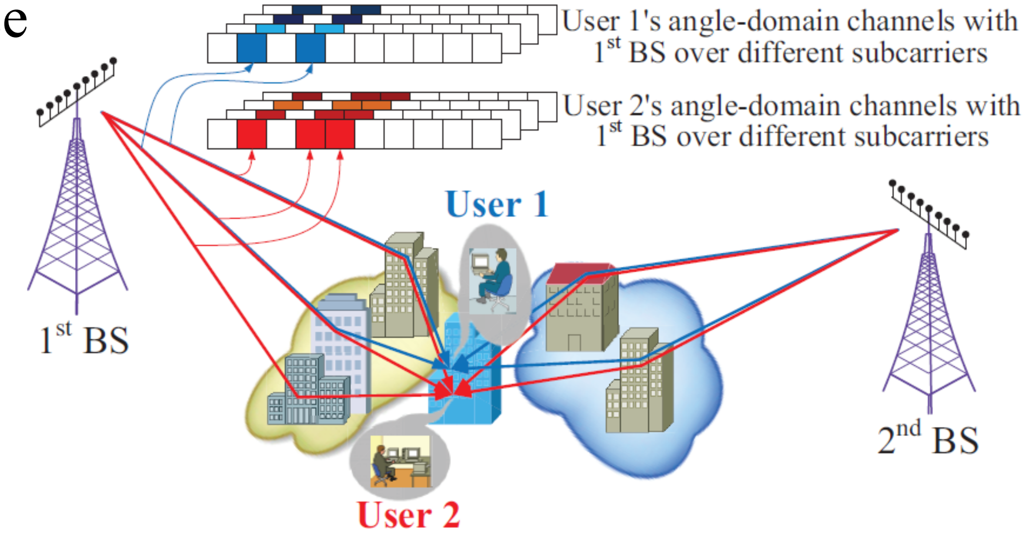


Fig. 3. Illustration of the angle-domain sparsity of massive MIMO channels.

CONTENTS

3. Proposed CS-Based CSIT Estimation Scheme

A. Pilot Training for CSIT Estimation in Multi-Cell Scenario

- The proposed scheme considers the downlink CSIT estimation and uplink feedback[8]. For the kth user of the central target cell ($l=0$) in the tth time slot, the received pilot signal fed back to the BS can be written as

$$\begin{aligned} r_{k,p}^{t} &= \sum_{l=0}^{L-1} \left(\mathbf{s}_{l,p}^{t}\right)^{\mathrm{T}} \mathbf{h}_{k,l,p} + w_{k,p}^{t} \\ &= \sum_{l=0}^{L-1} \left(\mathbf{s}_{l,p}^{t}\right)^{\mathrm{T}} \mathbf{h}_{k,l,p} \delta\left(\rho_{k,l} > \rho_{\mathrm{th}}\right) + \sum_{l=0}^{L-1} \left(\mathbf{s}_{l,p}^{t}\right)^{\mathrm{T}} \mathbf{h}_{k,l,p} \delta\left(\rho_{k,l} < \rho_{\mathrm{th}}\right) + w_{k,p}^{t} \qquad (5) \\ &= \sum_{l=0}^{L-1} \left(\mathbf{s}_{l,p}^{t}\right)^{\mathrm{T}} \mathbf{h}_{k,l,p} \delta\left(\rho_{k,l} > \rho_{\mathrm{th}}\right) + \tilde{w}_{k,p}^{t}, \end{aligned}$$

where $\delta(\bullet)$ is Dirac delta function, $\mathbf{s}_{l,p}^{t}$ is the downlink pilot, ρ_{th} is a predefined signal-to-noise-ratio (SNR) threshold, $\rho_{k,l}$ is the SNR, $\tilde{w}_{k,p}^{t}$ is the effective noise.

3. Proposed CS-Based CSIT Estimation Scheme

A. Pilot Training for CSIT Estimation in Multi-Cell Scenario

- Due to the angle-domain sparsity, (5) can be rewritten as

$$\begin{aligned} r^t_{k,p} &= \sum_{l\in\Pi_k} \left(\mathbf{s}^t_{l,p}\right)^{\mathrm{T}} \mathbf{F}\tilde{\mathbf{h}}_{k,l,p} + \tilde{w}^t_{k,p} = \sum_{l\in\Pi_k} \phi^t_{l,p}\tilde{\mathbf{h}}_{k,l,p} + \tilde{w}^t_{k,p} \\ &= \boldsymbol{\theta}^t_{k,p}\bar{\tilde{\mathbf{h}}}_{k,p} + \tilde{w}^t_{k,p}, \end{aligned} \tag{6}$$

where

$$\begin{cases} \Pi_k = \left\{ l : \rho_{k,l} > \rho_{\mathrm{th}}, 0 \le l \le L-1 \right\}, \\ \phi^t_{l,p} = \left(\mathbf{s}^t_{l,p}\right)^{\mathrm{T}} \mathbf{F} \in \mathbb{C}^{1\times M}, \\ \boldsymbol{\theta}^t_{k,p} = \left[\phi^t_{\Pi_k(1),p}, \phi^t_{\Pi_k(2),p}, \cdots, \phi^t_{\Pi_k\left(|\Pi_k|_c\right),\mathrm{p}} \right] \in \mathbb{C}^{1\times M|\Pi_k|_c}, \\ \bar{\tilde{\mathbf{h}}}_{k,p} = \left[\bar{\tilde{\mathbf{h}}}^{\mathrm{T}}_{k,\Pi_k(1),p}, \bar{\tilde{\mathbf{h}}}^{\mathrm{T}}_{k,\Pi_k(2),p}, \cdots, \bar{\tilde{\mathbf{h}}}^{\mathrm{T}}_{k,\Pi_k\left(|\Pi_k|_c\right),p} \right]^{\mathrm{T}} \in \mathbb{C}^{M|\Pi_k|_c\times 1}. \end{cases} \tag{7}$$

3. Proposed CS-Based CSIT Estimation Scheme

A. Pilot Training for CSIT Estimation in Multi-Cell Scenario

- Moreover, the channel is considered to be unchanged in G successive OFDM symbols with the channel cohere time[5]. Therefore, we jointly collecting the feedback pilots in G successive OFDM symbols and obtain

$$\mathbf{r}_{k,p}^{[G]} = \boldsymbol{\Theta}_{k,p}^{[G]} \bar{\tilde{\mathbf{h}}}_{k,p} + \mathbf{w}_{k,p}^{[G]}, \tag{8}$$

where

$$\begin{cases} \mathbf{r}_{k,p}^{[G]} = \left[\left(r_{k,p}^{1} \right)^{\mathrm{T}}, \left(r_{k,p}^{2} \right)^{\mathrm{T}}, \cdots, \left(r_{k,p}^{G} \right)^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{G\times 1}, \\ \boldsymbol{\Theta}_{k,p}^{[G]} = \left[\left(\boldsymbol{\theta}_{k,p}^{1} \right)^{\mathrm{T}}, \left(\boldsymbol{\theta}_{k,p}^{2} \right)^{\mathrm{T}}, \cdots, \left(\boldsymbol{\theta}_{k,p}^{G} \right)^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{G\times M \left| \Pi_k \right|_c}, \\ \mathbf{w}_{k,p}^{[G]} = \left[\tilde{w}_{k,p}^{1}, \tilde{w}_{k,p}^{2}, \cdots, \tilde{w}_{k,p}^{G} \right]^{\mathrm{T}} \in \mathbb{C}^{G\times 1}. \end{cases} \tag{9}$$

3. Proposed CS-Based CSIT Estimation Scheme

B. CS-Based Estimation Algorithm

- To estimate $\bar{\tilde{\mathbf{h}}}_{k,p}$ from (8), the conventional algorithms usually requires $G \geq M\left|\Pi_k\right|_c$ to obtain reliable performance[4].
- Fortunately, the angular sparsity can help reduce the training overhead according to the CS theory. Specifically, we consider the partially common support shared by K users

$$\mathbf{R}_p^{[G]} = \mathbf{\Theta}_p^{[G]} \bar{\tilde{\mathbf{H}}}_p + \mathbf{W}_p^{[G]}, 1 \leq p \leq P, \tag{10}$$

where

$$\begin{cases} \mathbf{R}_p^{[G]} = \left[\mathbf{r}_{1,p}^{[G]}, \mathbf{r}_{2,p}^{[G]}, \cdots, \mathbf{r}_{K,p}^{[G]}\right]^{\mathrm{T}} \in \mathbb{C}^{G \times K}, \\ \Pi_1 = \Pi_2 = \cdots = \Pi_K = \Pi, \\ \mathbf{\Theta}_{1,p}^{[G]} = \mathbf{\Theta}_{2,p}^{[G]} = \cdots = \mathbf{\Theta}_{K,p}^{[G]} = \mathbf{\Theta}_p^{[G]} \in \mathbb{C}^{G \times M\left|\Pi_k\right|_c}. \end{cases} \tag{11}$$

3. Proposed CS-Based CSIT Estimation Scheme

B. CS-Based Estimation Algorithm

- Given the measurements (10) and the sparse constraints (3) and (4), the CSI matrix $\{\bar{\tilde{\mathbf{H}}}_p\}_{p=1}^{P}$ can be required by solving the following optimization problem

$$\min_{\bar{\tilde{\mathbf{H}}}_p, 1\le p\le P} \sum_{p=1}^{P}\left(\sum_{k=1}^{K}\left\|\bar{\tilde{\mathbf{h}}}_p\right\|_0^2\right)^{1/2}$$

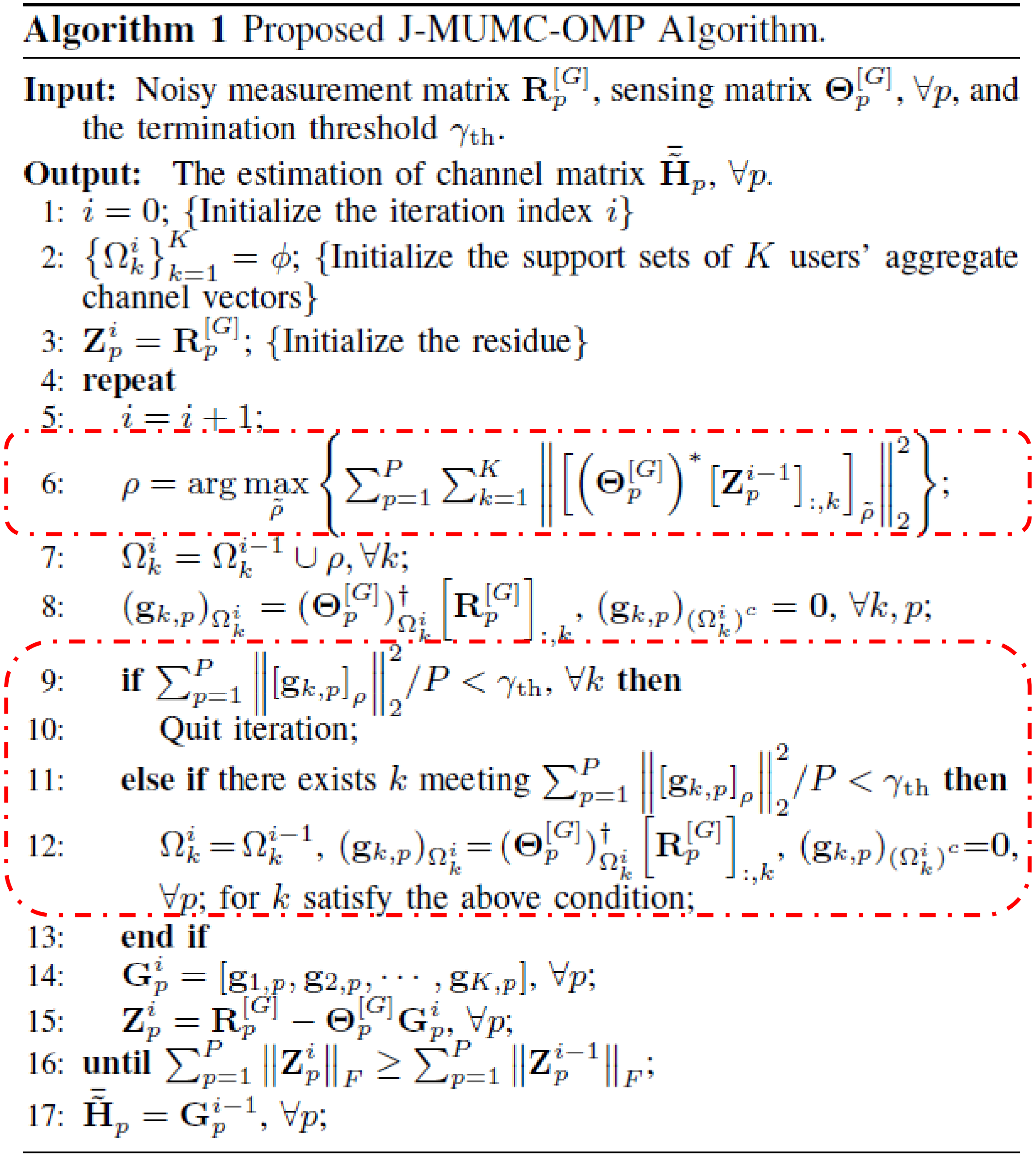
$$\text{s.t. } \mathbf{R}_p^{[G]} = \mathbf{\Theta}_p^{[G]}\bar{\tilde{\mathbf{H}}}_p, \Omega_{k,l,p} = \Omega_{k,l}, \forall p, \bigcap_{k=1}^{K}\Omega_{k,l} \neq \varnothing. \qquad (12)$$

- To solve (12), we extend the orthogonal matching pursuit (OMP) algorithm to a joint multi-user multi-carrier OMP (J-MUMC-OMP) algorithm, which leverages the common sparsity among multi-carrier and the partially common sparsity shared by users in a group.

3. Proposed CS-Based CSIT Estimation Scheme

B. CS-Based Estimation Algorithm

- Line 6 leverages the sparsity constraints (3) and (4);
- Line 9-10 avoid the over-estimation of sparsity;
- Line 11-12 consider the case that only a part of K users have ρth non-zero element in angle-domain channel vector.



Algorithm 1 Proposed J-MUMC-OMP Algorithm.

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Input: Noisy measurement matrix $\mathbf{R}_p^{[G]}$, sensing matrix $\mathbf{\Theta}_p^{[G]}$, $\forall p$, and
    the termination threshold $\gamma_{\rm th}$.
Output: The estimation of channel matrix $\tilde{\bar{\mathbf{H}}}_p$, $\forall p$.
1: $i = 0$; {Initialize the iteration index $i$}
2: $\{\Omega_k^i\}_{k=1}^K = \phi$; {Initialize the support sets of $K$ users' aggregate
    channel vectors}
3: $\mathbf{Z}_p^i = \mathbf{R}_p^{[G]}$; {Initialize the residue}
4: repeat
5:     $i = i + 1$;
6:     $\rho = \arg\max_{\tilde{\rho}} \left\{ \sum_{p=1}^P \sum_{k=1}^K \left\| \left[ \left( \mathbf{\Theta}_p^{[G]} \right)^* [\mathbf{Z}_p^{i-1}]_{:,k} \right]_{\tilde{\rho}} \right\|_2^2 \right\}$;
7:     $\Omega_k^i = \Omega_k^{i-1} \cup \rho, \forall k$;
8:     $(\mathbf{g}_{k,p})_{\Omega_k^i} = (\mathbf{\Theta}_p^{[G]})^\dagger_{\Omega_k^i} \left[ \mathbf{R}_p^{[G]} \right]_{:,k}$, $(\mathbf{g}_{k,p})_{(\Omega_k^i)^c} = \mathbf{0}$, $\forall k, p$;
9:     if $\sum_{p=1}^P \left\| [\mathbf{g}_{k,p}]_\rho \right\|_2^2 / P < \gamma_{\rm th}$, $\forall k$ then
10:        Quit iteration;
11:    else if there exists $k$ meeting $\sum_{p=1}^P \left\| [\mathbf{g}_{k,p}]_\rho \right\|_2^2 / P < \gamma_{\rm th}$ then
12:        $\Omega_k^i = \Omega_k^{i-1}$, $(\mathbf{g}_{k,p})_{\Omega_k^i} = (\mathbf{\Theta}_p^{[G]})^\dagger_{\Omega_k^i} \left[ \mathbf{R}_p^{[G]} \right]_{:,k}$, $(\mathbf{g}_{k,p})_{(\Omega_k^i)^c} = 0$,
           $\forall p$; for $k$ satisfy the above condition;
13:    end if
14:    $\mathbf{G}_p^i = [\mathbf{g}_{1,p}, \mathbf{g}_{2,p}, \cdots, \mathbf{g}_{K,p}]$, $\forall p$;
15:    $\mathbf{Z}_p^i = \mathbf{R}_p^{[G]} - \mathbf{\Theta}_p^{[G]} \mathbf{G}_p^i$, $\forall p$;
16: until $\sum_{p=1}^P \left\| \mathbf{Z}_p^i \right\|_F \geq \sum_{p=1}^P \left\| \mathbf{Z}_p^{i-1} \right\|_F$;
17: $\tilde{\bar{\mathbf{H}}}_p = \mathbf{G}_p^{i-1}$, $\forall p$;
```

3. Proposed CS-Based CSIT Estimation Scheme

C. CS-Based Downlink Pilot Design in Multi-Cell Scenario

- The design of measurement matrices $\boldsymbol{\Theta}_p^{[G]}$ for different p in (12) are important to ensure the reliable channel estimation in CS theory.
- Reviewing the constitution of $\boldsymbol{\Theta}_p^{[G]}$, it can be observed that $\boldsymbol{\Theta}_p^{[G]}, \forall p$ are only determined by the pilot signals $\{\mathbf{s}_{l,p}^t\}_{l=0,p=1,t=1}^{L-1,P,G}$.
- According to [13], we consider each element of pilot signals can be off-line designed as

$$\left[\mathbf{s}_{l,p}^t\right]_m = e^{j\theta_{m,l,p,t}}, 1 \le m \le M, 1 \le t \le G, 1 \le p \le P, 0 \le l \le L-1, \qquad (13)$$

where $\theta_{m,l,p,t}$ follows the i.i.d uniform distribution in $\left[0, 2\pi\right)$.

3. Proposed CS-Based CSIT Estimation Scheme

D. Multi-Cell Joint Precoding

- The proposed scheme can use the low training overhead to estimate CSIT, which can be leveraged to perform multi-cell joint precoding to combat ICI.
- Specifically, we consider
  - 1) each BS uses zero forcing (ZF) precoding to serve multiple users;
  - 2) multiple users served by the BS using the same time-frequency resource should come from different user groups to reduce the correlation of different users channel vectors and enhance the system capacity;
  - 3) each user is jointly served by multiple adjacent BSs according to the channel quality.

CONTENTS

4. Simulation Results

Important Simulation Parameters

the number of hexagonal cells L	7
the number of antennas in each BS M	128
the size of OFDM symbol P	50
the cell radius	1 km
the path loss	$\beta_{\mathrm{PL}} = d^{-\alpha}$
the predefined SNR threshold ρ_{th}	3/5/10/10/10 for $\rho_{\mathrm{edge}} =$ 10/15/20/25/30 dB
the termination threshold γ_{th}	0.006/0.004/0.002/0.004/0.003 for $\rho_{\mathrm{edge}} =$ 10/15/20/25/30 dB

4. Simulation Results

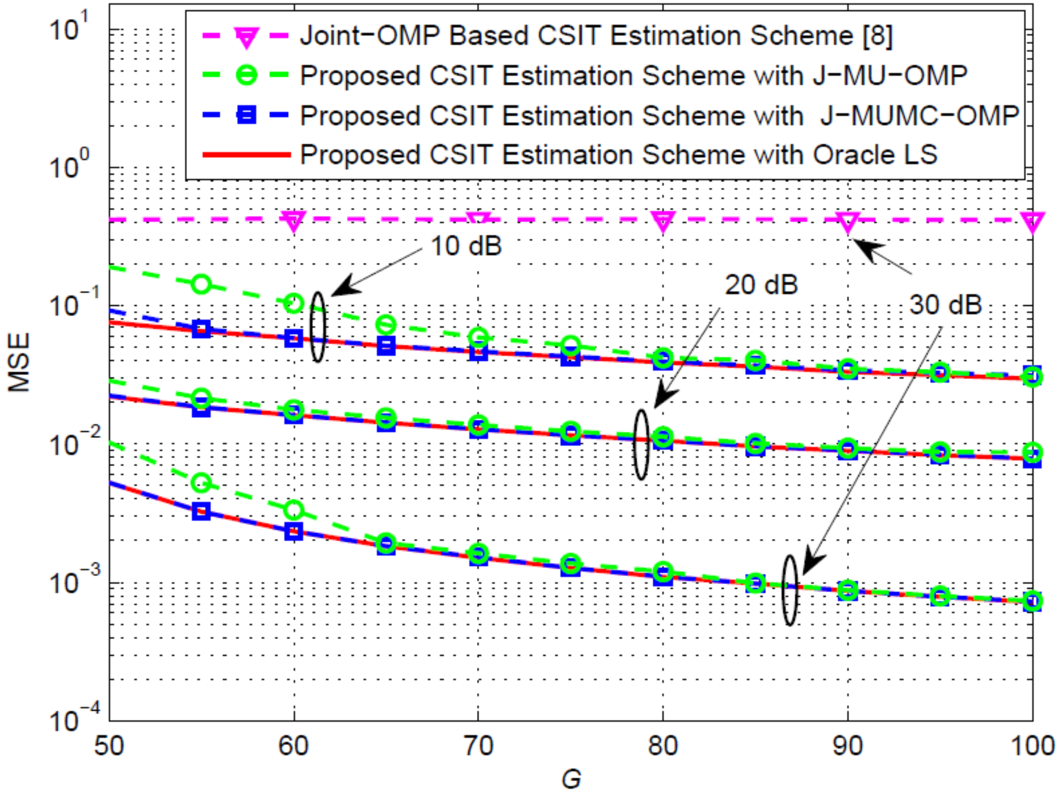

Fig. 4. Comparison of channel estimation MSE performance of different CSIT estimation solutions versus G at different ρ_{edge}, $K = 10$.

4. Simulation Results

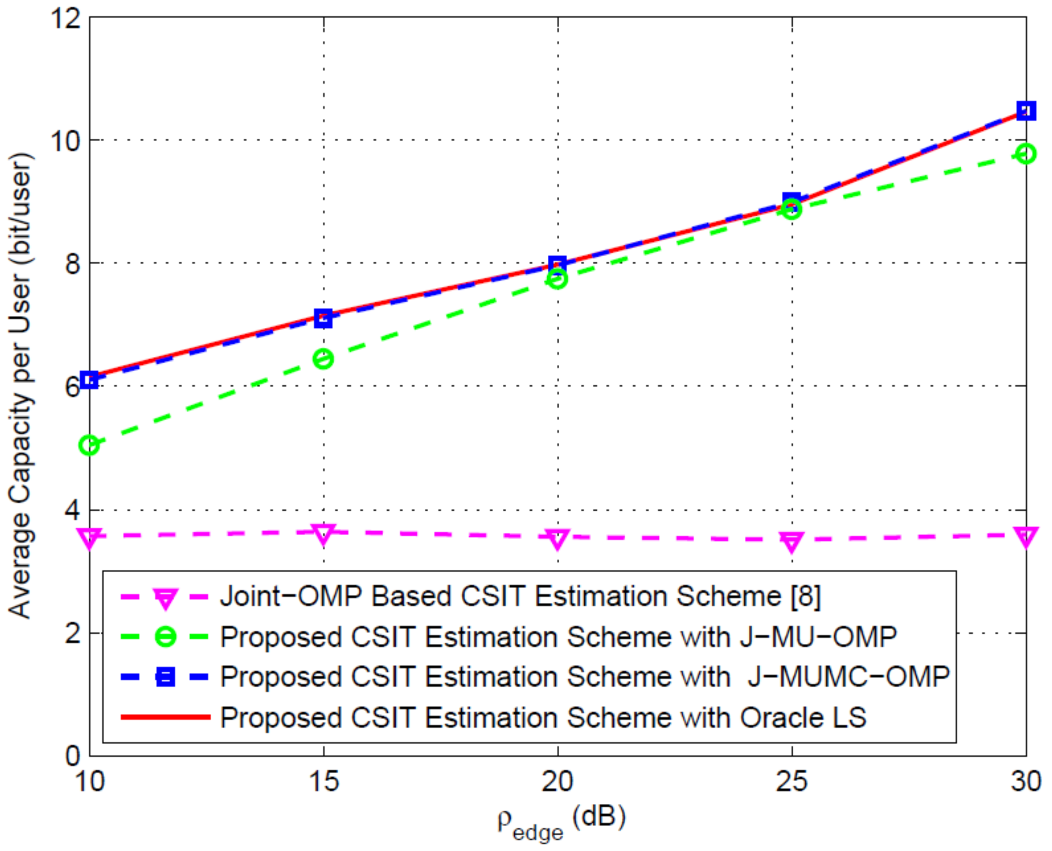

Fig. 5. Comparison of downlink average throughput per user with multi-cell joint ZF precoding when $G = 55$.

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Thanks for your attention!